
Mathematics Specialist

Unit Four

4

UNIT FOUR PRELIMINARY WORK

Having reached this stage of the book it is naturally assumed that you are already familiar with the work of the previous chapters (as well as Units One and Two of the *Mathematics Specialist* course and Units One and Two of the *Mathematics Methods* course). It is also assumed that you are familiar with the content of Unit Three of the *Mathematics Methods* course.

In particular, for this unit, familiarity with the following concepts will be assumed:

Differentiation

- Second and higher-order derivatives.
- The product, quotient and chain rules.
- Applications to curve sketching.
- Rates of change.
- Application to rectilinear motion.
- Optimisation.
- Small changes and marginal rates of change.
- Differentiation of e^x and $e^{f(x)}$.
- Differentiation of trigonometric functions.

Antidifferentiation and Integration

- Antidifferentiating algebraic and trigonometric functions.
- Antidifferentiating expressions involving e^x and $e^{f(x)}$.
- The fundamental theorem of calculus.
- Indefinite and definite integrals.
- Area under and between curves.
- Application to rectilinear motion.
- Total change from rate of change.

Statistics

It will also be assumed that the statistical concepts of the measures of central tendency (mean and median) and of dispersion (range and standard deviation) are familiar concepts.

Trigonometric identities

From your study of earlier *Mathematics Specialist* units, and of units from *Mathematics Methods*, it is assumed that you are already familiar with the following trigonometrical identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

from which it follows that
and

$$\begin{aligned}\tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \operatorname{cosec}^2 \theta\end{aligned}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

from which it follows that

$$\sin 2A = 2 \sin A \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

from which it follows that

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2 \sin^2 A \\ &= 2 \cos^2 A - 1\end{aligned}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

from which it follows that

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

And the following rules for the products of sines and cosines:

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\sin P + \sin Q = 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$\sin P - \sin Q = 2 \cos\left(\frac{P+Q}{2}\right) \sin\left(\frac{P-Q}{2}\right)$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\cos P + \cos Q = 2 \cos\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\cos P - \cos Q = -2 \sin\left(\frac{P+Q}{2}\right) \sin\left(\frac{P-Q}{2}\right)$$

Limit of a sum

It is assumed that your familiarity with the fundamental theorem of calculus means that you understand that we can write the **limit of a sum**

$$\lim_{\delta x \rightarrow 0} \sum_{x=a}^{x=b} f(x) \delta x$$

as the **definite integral**

$$\int_a^b f(x) dx,$$

and that this definite integral can be evaluated using **antidifferentiation**.

Use of technology

As with the previous unit you are encouraged to use your calculator, computer programs and the internet whenever appropriate during this unit.

However, whilst familiarity with these technologies is assumed, you should make sure that you can also perform the basic processes without the assistance of such technology when required to do so.

Note

The illustrations of calculator displays shown in the book may not exactly match the display from your calculator. The illustrations are not meant to show you exactly what your calculator will necessarily display but are included more to inform you that at that moment the use of a calculator could well be appropriate.



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